Image Resizing and Rotation Based on the Consistent Resampling Theory

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***Abstract*— The recently proposed consistent resampling theory for non-bandlimited signals is applied to image resizing and rotation. Images with high frequency components can be resampled using this scheme to achieve high quality performance. Image resizing is treated as resampling using non-ideal interpolation functions. Both zoom in and zoom out by non-integer factors are considered. Image rotation is also formulated as a resampling process. We show that our approach outperforms other linear image processing techniques without increasing the computational cost.**

I. Introduction

Resizing and rotation are common image processing operations. In each case the original image is resampled to compute the new pixel values in the resulting image. The resampling process involves two steps [1]. First, the discrete input is interpolated to a continuous signal. Then, this continuous signal is resampled at the desired locations and at the desired sampling rate to produce a discrete output.

The performance of the resampling process is conventionally indicated by the mean squared error (MSE) of its embedded interpolation process. Therefore, the focus has been on improving the performance of the interpolation process [2]. Some non-linear methods have been devised to minimize the MSE. In [3], the signal is expressed in terms of the interpolation function. The optimally reconstructed signal is obtained by solving a set of separable partial differential equations such that the MSE is minimized. A similar approach can be found in the work on scalable video coding [4]. The upsampling and downsampling of the discrete signals are modeled and solved via differential equations. Adaptive interpolating filters are used in [5] to minimize the MSE for each individual estimated sample. A generalized approach is proposed in [6] that makes use of kernel regression methods to obtain the coefficients of the spline functions that minimize the MSE.

The development of consistent sampling theory provides a linear method to achieve minimum MSE when noise is not present [7], [8]. It simplifies the design process and reduces computational complexity without compromising performance. Consequently, the theory is then directly applied to image resizing and rotation [9], [10]. Two major techniques, oblique interpolation [11] and quasi interpolation [12], have been developed recently based on the principle of consistent sampling which has been shown to provide better performance than conventional techniques.

In [13], the techniques derived for consistent sampling are used to analyze the performance of video de-interlacing. In this case, results contradictory to common sense are obtained. What this reveals is that optimal *resampling* cannot be obtained from optimal *sampling* for non-bandlimited signals. Consequently, while consistent sampling is optimal for sampling without noise, oblique and quasi interpolation methods cannot guarantee that optimal resampling can be achieved.

Recently, the *consistent resampling theory* (CRT) has been proposed to resample signals with non-bandlimited response in an optimal way [14]. When used to evaluate the performance of de-interlacing, CRT produces consistent result with the intuitive answer, which can not be achieved using consistent sampling theory. In this paper, we apply CRT to image resizing and rotation. A digital filter is designed to implement the correction filter derived by CRT. We show by experimental results that CRT outperforms existing linear methods without increasing computational cost.

II. Consistent Resampling Theory

The consistent sampling theory developed in [7] proposed a new criteria to compare continuous signals when the sampling and interpolation function are non-ideal, as is often the case in image processing. CRT resembles the consistent sampling theory. But it is more than a simple extension since measuring the difference between the input and output is very different for a resampling system compared to a sampling/reconstruction system.

CRT proposes that resampling is *consistent* if the output discrete signal appears to be the same as the input discrete signal as far as the interpolation function is concerned. In

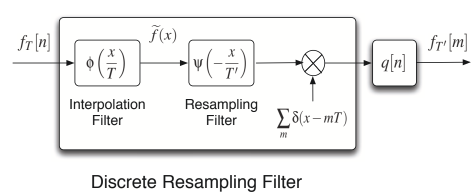


Fig. 1. The consistent resampling system with the correction filter.

other words, to the interpolation function, both input and output signals describe the same analog signal. In general, for any arbitrary ϕ and ψ, a resampling system will not be *consistent*. However, a digital correction filter $q[n]$ can be incorporated into the system in the way shown in Fig. 1 to achieve consistent resampling. The following proposition provides a formula for the design of this correction filter.

Proposition 1: Let

$$C_{\phi\psi}(\omega) = \sum_{m,n} c_{\phi\psi}[n,m] e^{j\omega(m-nT_r)} \tag{1}$$

be the frequency response of the sampled cross correlation $\{c_{\phi\psi}[n,m]\}_{m,n\in\mathbb{Z}}$ of $\phi(\frac{x}{T})$ and $\psi(\frac{x}{T'})$ where

$$c_{\phi\psi}[n,m] = \int_x \phi\left(\frac{x}{T} - n\right) \psi\left(\frac{x}{T'} - m\right) dx \tag{2}$$

with $T_r = T/T'$. Similarly, let $C_{\phi\phi_d}(\omega)$ be that of $\{c_{\phi\phi_d}[n,m]\}_{m,n\in\mathbb{Z}}$, the sampled cross correlation of $\phi(\frac{x}{T})$ and $\phi_d(\frac{x}{T'})$, the dual operator of $\phi(\frac{x}{T'})$, given by

$$c_{\phi\phi_d}[n,m] = \int_x \phi\left(\frac{x}{T} - n\right) \phi_d\left(\frac{x}{T'} - m\right) dx \tag{3}$$

Then the resampling system in Figure 1 is consistent if the frequency response of the digital correction filter is

$$Q(\omega) = \frac{C_{\phi\phi_d}(\omega)}{C_{\phi\psi}(\omega)} \tag{4}$$

Since $T_r = T/T'$ is generally not an integer, $(m - nT_r)$ is also typically a non-integer. Therefore, the conventional impulse invariant approach cannot be used to obtain the impulse response implemented by ideal sampling its continuous counterpart which enforces consistent resampling. The following proposition derives the formula for the continuous filter.

Proposition 2: Replace $q[n]$ in Fig. 1 by a continuous filter $q(x)$. Resampling is consistent if the frequency response of correction filter $q(x)$ satisfies:

$$Q(\Omega) = \frac{\Phi_d(\Omega)}{\Psi(\Omega)} \tag{5}$$

where $\phi_d(x)$ is the dual operator of ϕ.

III. Image Resizing

Now we consider the application of consistent resampling to image resizing. In our experiments, an image is either enlarged by a factor of 1.25 or reduced to 0.8 of its size. These factors are chosen arbitrarily and any other factors could have been chosen instead.

Four different resampling techniques are considered. They are: (1) classic interpolation; (2) oblique interpolation [11]; (3) quasi interpolation [12] and (4) consistent resampling. In order to obtain a fair comparison, the interpolating function used by all four techniques is the first order B-spline β^1 where $\beta^1(x) = 1$ for $x \in [-1, 1]$ and 0 otherwise. We also set the length of the correction filter in all cases to be 3.

The correction filter for consistent resampling is obtained by (4) in Proposition 1. It is implemented by sampling the continuous correction filter $q(x)$ defined by Proposition 2 at rate T'. Since $\psi(x) = \delta(x)$ and $\Psi(\Omega) = 1$, from (5), $Q(\Omega) = \Phi_d(\Omega)$. Therefore

$$Q(\Omega) = B_d^1(\Omega) = \frac{sinc^2\left(\frac{\Omega}{2}\right)}{1 - \frac{2}{3}\sin^2\left(\frac{\Omega}{2}\right)} \tag{6}$$

To sample $q(x)$ at T', the frequency response of the sequence $\{q[nT']\}_{n\in\mathbb{Z}}$, $Q(\omega)$ is related to $Q(\Omega)$ by setting $\Omega = \frac{\omega}{T'}$, therefore

$$Q(\omega) = Q(\Omega)|_{\Omega=\omega/T'} = \frac{sinc^2\left(\frac{\omega}{2T'}\right)}{1 - \frac{2}{3}\sin^2\left(\frac{\omega}{2T'}\right)} \tag{7}$$

For digital correction filter $q[n]$ of length 3, we have

$$\begin{aligned} Q(\omega) &= [c_{-1}e^{j\omega} + c_0 + c_1 e^{-j\omega}]^{-1} \\ &= \left[c_0 + c_1(e^{j\omega} + e^{-j\omega})^{-1}\right]^{-1} \end{aligned} \tag{8}$$

for some constants c_0 and $c_1 = c_{-1}$. Using Taylor's expansion to express (7) and set (7) = (8), when $T' = 0.8$, it can be worked out that $b = \frac{4}{75}$ and $a = \frac{67}{75}$. Therefore $Q(\omega)$ is given by

$$Q(\omega) = \frac{75/4}{e^{j\omega} + 67/4 + e^{-j\omega}} \tag{9}$$

For zooming out to 0.8 of its size, $T = 1$ and $T' = 1.25$. The correction filter is given by

$$Q(\omega) = \frac{592/25}{e^{j\omega} + 542/25 + e^{-j\omega}} \tag{10}$$

Note that in both cases $q[n]$ are IIR filters with symmetric structure. Such filter can be easily implemented using the techniques developed in [15]. The "Rays" image which is an artificial image with high frequency components is used in our experiment, as shown in Fig. 2. The image is enlarged by a factor of 1.25 eight consecutive times. Subsequently, the enlarged image is reduced to 0.8 of its size eight consecutive times so the resulting image has the same size as the original. Fig. 3 shows the resulting images.

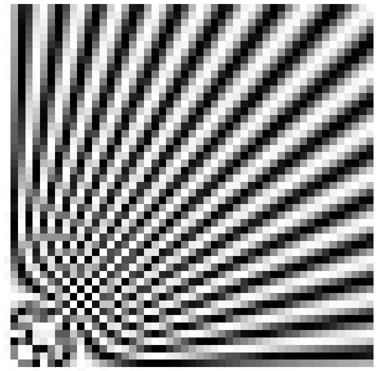
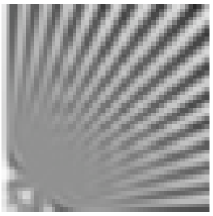
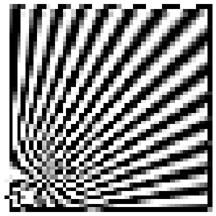


Fig. 2. The original Rays image.

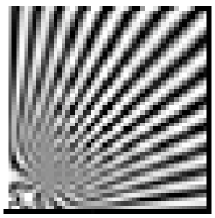
When an image is zoomed in and out several times, artifacts are created due to aliasing and blurring, especially for the high frequency components. The Rays image contains mainly high frequency components. Using classic interpolation, the



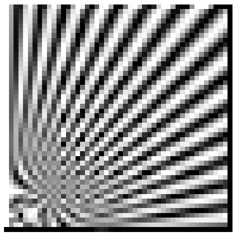
(a) Classic Interpolation



(b) Oblique Interpolation



(c) Quasi Interpolation



(d) Consistent Resampling

Fig. 3. The Rays image after eight consecutive enlargements followed by eight consecutive reductions.

TABLE I
PSNR FOR IMAGE ZOOMED OUT BY 1.25 FOR 8 CONSECUTIVE TIMES, FOLLOWED BY ZOOMED IN BY 0.8 FOR 8 CONSECUTIVE TIMES.

PSNR	Rays	Lena	Pepper	Head
Classic	17.24	59.20	54.17	42.59
Oblique	24.71	61.21	54.62	50.58
Quasi	23.87	65.93	55.25	53.20
Consistent	29.96	66.01	56.17	55.64

details are completely missing near the lower left corner (see Fig. 3(a)). The details are better preserved by oblique interpolation as can be observed from Fig. 3(b). However, the effect of overshoot, i.e. increased contrast, is particularly evident near the borders of the image. The quasi interpolation method does not preserve the high frequency components as well as oblique interpolation. As shown in Fig. 3(c), on one hand, details at the left lower corner has partly disappeared and the check pattern is invisible. However, the blurred area is significant less than that obtained by classic interpolation.

Consistent resampling outperforms the other three techniques in preserving high frequency components. As shown in Fig. 3(d), the check pattern is well recognizable. The contrast and intensity of the image is unchanged as well. Other images are tested as well and the PSNR is recorded in Table I. It can be observed that consistent resampling produces the best visual results and highest PSNR among the four techniques considered.

Note that a 3-tap correction filter is used for consistent resampling in order to make the comparisons fair. If a higher order filter is used, then the high frequency components of the images will be even better preserved by consistent resampling. Fig. 4 shows the result obtained by using a 5-tap correction filter on the Rays image. The resultant PSNR is 43.22dB which more than doubled the improvement made by a 3-tap filter over the classic technique.

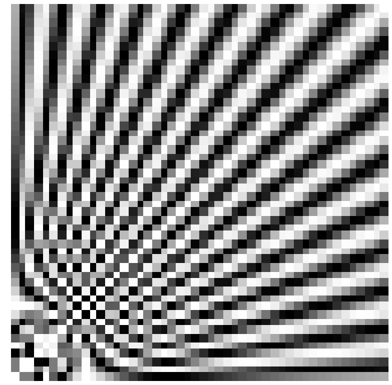


Fig. 4. Using a 5th order correction filter on the Rays image.

IV. IMAGE ROTATION

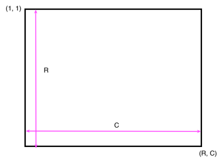
Image rotation by an angle θ anti-clockwise is usually performed by multiplying the image with the rotation matrix $R(\theta)$ [16]

$$R(\theta) = \underbrace{\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1 & 0 \\ \sin\theta & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ 0 & 1 \end{bmatrix}}_{A}$$

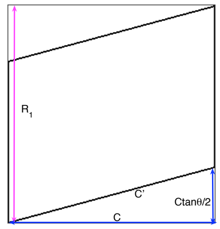
Thus the multiplication with the rotation matrix $R(\theta)$ can be separated into three sequential steps – multiplication by matrices A, B and A. For a pixel at coordinates (m, n) in the original image, after the first step its new coordinates (m', n') are given by $m' = m - n\tan\frac{\theta}{2}$ and $n' = n$, that is, the row index m is translated by $-n\tan\theta/2$ while the column index n is unchanged. This is a 1-D process. Similarly, in the second step, multiplication by matrix B leaves the row index unchanged while the column index is translated by $m\sin\theta$. Thus the whole transformation process can be decomposed into a sequence of 1-D translations, as shown in Fig. 5.

Existing methods for rotation based on this three-step process are focused on the design of appropriate translation algorithms [16]. We interpret the decomposed rotation process from a new angle. Assume that the size of the image is $R \times C$ pixels as shown in Fig. 5(a). After the first step, each column is translated and so the image becomes what is shown in Fig. 5(b). Therefore, each row in the original image is effectively resized by a the factor of $L_1 = C'/C = \sqrt{1 + \tan^2\theta/2}$. Assuming that the sampling period of the original signal is $T = 1$, the resampling period is given by $T' = 1/L_1$. Similarly, in the second step each column is resized from R_1 to R_1' as shown in Fig. 5(c). The resizing factor is $L_2 = R_1'/R_1 = \sqrt{1 + \sin^2\theta}$ and the corresponding resampling period is $T' = 1/L_2$. In the third step, the columns of the image in Fig. 5(c) is translated in the same way as in the first step. The resizing factor is $L_1 = C_2'/C_2$.

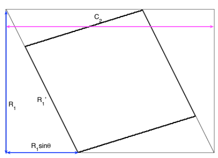
Since the rotation process has now been formulated as a sequence of resizing operations, we can make use of our



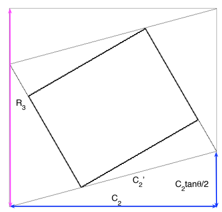
(a) Original Figure



(b) Step 1: Column-wise Translation



(c) Step 2: Row-wise Translation



(d) Step 3: Column-wise Translation

Fig. 5. Illustration of decomposed rotation process.

(a) Classic Interpolation

(b) By CRT

Fig. 6. After twelve rotations of 30° by classic resampling and consistent resampling.

consistent resampling system to perform the rotation. For a given interpolation function ϕ and the parameters T and T', a consistent correction filter can be designed similarly in Section III Note that resizing factors L_1 and L_2 are functions of θ only and does not depend on the size of the image, the correction filters are applicable to images of any size.

The image Boat is used to test our result. It is rotated 30° anti-clockwise twelve times. Fig. 6(a) shows the results obtained using the conventional method for rotation as implemented by the "imrotate" function in MATLAB. The interpolation method chosen is 'bilinear'. For a fair comparison, we use β^1 as the interpolation function for consistent resampling. Fig. 6(b) shows the results obtained using correction filtering. It is obvious that the fine details of the image are highly preserved since consistent resampling does not assume a bandlimited signal.

Four other images – Lena, Barbara, Baboon and Camera, are tested as well and the resultant PSNR are recorded in Table II.

TABLE II

PSNR (dB) after 12 rotations of 30°.

	Lenna	Barbara	Baboon	Boat	Camera
Classic	57.9	45.97	39.55	49.83	46.82
Consistent	64.53	54.16	49.07	60.08	51.26

V. Conclusion

In this paper we demonstrate the practical use of CRT to image resizing and rotation. We designed IIR filters which approximate the desired response of the optimal CRT solution. The improvement in performance in comparison to the direct applications of consistent sampling theory to image processing is demonstrated through our experimental results.

Our consistent resampling approach to image resizing and rotation is simple and flexible. It simply involves computing the resampling factor and then obtaining the correction filter based on the interpolation function chosen. The computational complexity grows linearly with the size of the image and the order of the correction filter, which is the same as other linear algorithms.

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